VANILLA EQUITY OPTION PRICING OFFERED BY RISKWORX

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Typically European equity options are priced using the Black-Scholes model (Black & Scholes 1973) or that model adjusted for dividends by calculating a continuous dividend yield. This has the effect of spreading the dividend payment throughout the life of the option. In the case of several dividend payments, this is a satisfactory solution, for example, where the option is on an index (where the index is paying out several dividends, spread out through the period of optionality).

Thus, for European equity options where the underlying has no or several dividends, we will use the Black-Scholes formula. For American equity options with the underlying having no or several dividends, we may argue similarly. Here the approximation of Barone-Adesi and Whaley (Barone-Adesi & Whaley 1987) is popular, but we prefer the method of Bjerksund and Stensland (Bjerksund & Stensland 1993), (Bjerksund & Stensland 2002) as it is computationally far superior, and has been shown to be more accurate in long dated options. (Bjerksund & Stensland 2002) is a recent improvement over (Bjerksund & Stensland 1993).

Another standard approach (for the European case) is to reduce the stock value by the present value of dividends (the escrowed dividend method), or to increase the strike by the future value of dividends. Both are unsatisfactory approaches as they affect the stochastic process on the equity fairly significantly. See (Frishling 2002), (Bos & Vandermark September 2002), (Haug, Haug & Lewis 2003).

In the case of only a few dividend payments on the underlying equity, the original approach above - calculating a continuous dividend yield and using that in a closed form formula - is also no longer satisfactory, even for European options. The dividends occur at one or a few discrete times, but we are spreading them out throughout the life of the option by making this assumption, and this has a material effect on the stochastic process for the stock price.

This comment also applies to the classic binomial tree approach for pricing American options developed in (Cox, Ross & Rubinstein 1979). Use of a binomial tree necessitates that risk free rates are assumed constant, and that there is a constant dividend yield, as described above. This will lead to the same severe problems as before. Note that dividends cannot be made discrete in the tree approach because doing so will make the tree no longer recombine, which is computationally as disaster.

Much theory has been developed to price (European) options under the assumption that the dividends are a known proportion of the stock price on the dividend payment date. See (Björk 1998), for example. However, to use this approach alone is academic fiction: companies and brokers think of, predict, and eventually declare the reasonably short dated dividends in a currency unit. Furthermore, companies are very much loathe to reduce the dividend amount year on year, as a significant proportion of stock holders hold the stock purely for the purpose of receiving annuity revenue from the dividends (for example, retirees, who intend living on the dividends, and leaving the stock to their inheritors) and may transfer their holding to another stock if the dividend was decreased significantly (or even, was not keeping up with inflation). Thus, even if the stock price has decreased somewhat, the company will attempt to maintain dividend levels at more or less the same currency level, at least for a while. Thus, the model that dividends are a known proportion of share price is not practicable.

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Another more practical and meaningful possibility is that the first few dividends are known or predicted in cash, whilst the remaining dividends are predicted as a proportion of stock price.

In the case of an American call with one dividend, the formula of Roll, Geske, Whaley (Roll 1977), (Geske 1979), (Whaley 1981) is well known (amongst practitioners) to be arbitragable (and not so well known amongst software vendors, who often insist on offering this as the default model). Again, see (Frishling 2002), (Haug et al. 2003). Furthermore, their approach does not allow for the pricing of American puts (as is well known, the pricing of American puts is in general more difficult than the pricing of calls).

Thus, for European or American options with a few dividends, we will use a finite difference scheme for pricing. This finite difference scheme easily accommodates the discrete jumps of dividends.

One can use the finite difference approach for any number of dividends if prepared to input them. As the number of dividends increases, the benefits of these approaches are outweighed by the superior speed of using the continuous dividend yield proxy in the Black-Scholes or Bjerksund-Stensland formula.

On the PriceWorX input screen, one can choose between the following models:

- Black-Scholes - for European equity options where the underlying has no or several dividends. A dividend yield is calculated consistent with this information and is use in the Black-Scholes formula.
- Bjerksund and Stensland - for American equity options with the underlying having no or several dividends.
- Finite Differences - for European or American options with at least one but not many dividends, using finite differences.
- Analytic - the method of (Haug et al. 2003) for European options. Being phased out of the product, but currently retained for compatibility.

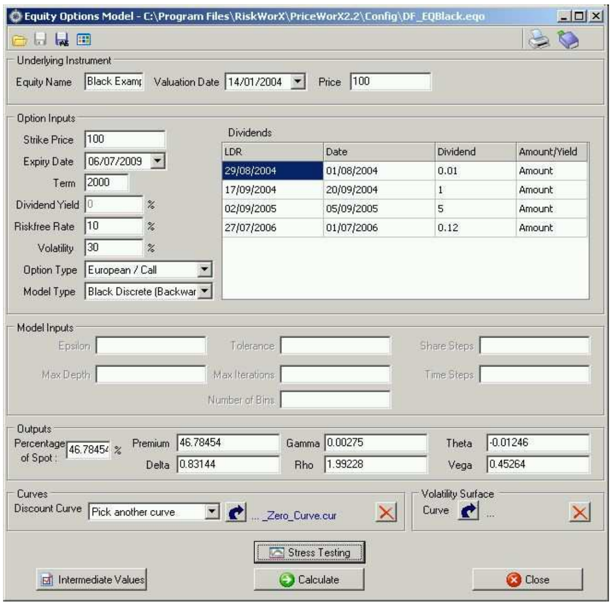

FIGURE 1. The input screen for equity options

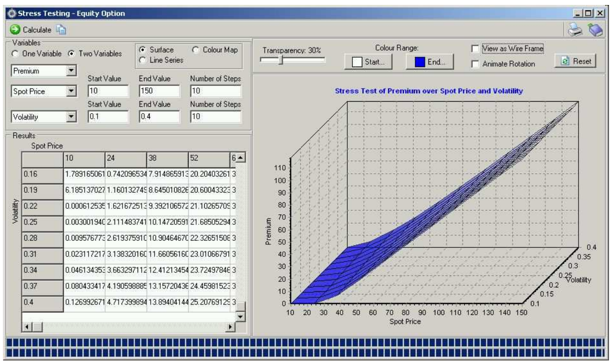

FIGURE 2. A matrix and display for profit and loss under stock and volatility movements

REFERENCES

Barone-Adesi, G. & Whaley, R. (1987), 'Efficient analytic approximation of American option values', *Journal of Finance* **42**(2), 301–320.

Bjerksund, P. & Stensland, G. (1993), 'Closed form approximation of American options', *Scandinavian Journal of Management* **9**, 87–99.

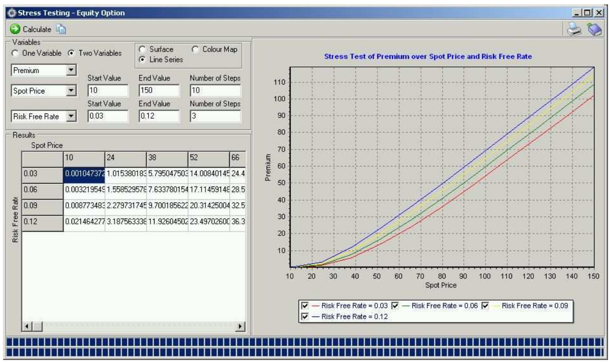

Figure 3. Profit and loss under different spot and interest rate scenarios

Bjerksund, P. & Stensland, G. (2002), 'Closed form valuation of American options'.
*http://www.nhh.no

Björk, T. (1998), *Arbitrage Theory in Continuous Time*, Oxford University Press.

Black, F. & Scholes, M. (1973), 'The pricing of options and corporate liabilities', *Journal of Political Economy* **73**(May-June), 637–659.

Bos, M. & Vandermark, S. (September 2002), 'Finessing fixed dividends', *Risk* **15**(9), 157–158.

Cox, J., Ross, S. & Rubinstein, M. (1979), 'Option pricing: a simplified approach', *Journal of Financial Economics* **7**, 229–263.

Frishling, V. (2002), 'A discrete question', *Risk* **January**(1).

Geske, R. (1979), 'A note on an analytic formula for unprotected American call options on stocks with known dividends', *Journal of Financial Economics* **7**(A), 375–380.

Haug, E. G., Haug, J. & Lewis, A. (2003), 'Back to basics: a new approach to the discrete dividend problem', *WILMOTT Magazine* **September**, 37–47.

Roll, R. (1977), 'An analytic valuation formula for unprotected American call options on stocks with known dividends', *Journal of Financial Economics* **5**, 251–258.

Whaley, R. (1981), 'On the valuation of American call options on stocks with known dividends', *Journal of Financial Economics* **9**, 207–211.

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