Bond Option Pricing – Clean or Dirty?

1. Bond Price Volatility  
2. Bond Option Pricing  
3. Conclusion
This note deals with the question of what the correct volatility inputs to bond option pricing in the South African market are. Some market players use price volatilities, whereas others use yield volatilities. Which is correct, if bonds are quoted on a yield basis in this market?

1. **Bond Price Volatility**

If one uses a model with ‘price’ as the input for the strike and spot price, we need to calculate a ‘price’ volatility. Since the SA bond market quotes bond prices in terms of yield, one cannot calculate an annual volatility for the yield and use this. The reason for this is that bond prices (both ‘clean’ and ‘dirty’) are non-linear functions of yield, and hence the ‘price’ volatility will not be the same as the ‘yield’ volatility.

**Approach 1 – Duration Conversion**

One approach for obtaining the ‘price’ volatility from a ‘yield’ volatility is by using the formula

\[ s = D y_0 s_y \]

where
- \( y_0 \) is the initial forward yield;
- \( D \) is the modified duration of the underlying bond; and
- \( s_y \) is the ‘yield’ volatility.

An advantage of this approach is that it takes into account the pull to par effect of the bond price, since as the underlying bond approaches maturity, the duration will decrease, and hence, so will the volatility.

**Approach 2 – Price Conversion**

This approach uses the yields to calculate bond prices, and from this calculates the ‘price’ volatility. The question then is whether to use ‘clean’ or ‘dirty’ prices for the volatility calculation.
A simple example will illustrate that it is necessary to use ‘clean’ prices. Consider a standard SA bond such as the R153. If the yield of this bond should remain unchanged at, say, 12% for the entire year, then it should be intuitively obvious that the volatility for the year should be 0%. However if we use ‘dirty’ prices, we obtain an annual volatility between 7% and 9%. This is due to the ‘saw tooth’ shape of ‘dirty’ bond prices over time. This is a result of the accrued interest component of the ‘dirty’ price. In Figure 1, it is clear on what dates the accrued interest is paid. ‘Clean’ prices are far smoother and produce a ‘price’ volatility that is close to 0%, as one would expect.

![Price Volatility with Constant Yield](image)

Figure 1

To further illustrate the point, we can consider actual market data and compare the 30-day exponentially weighted ‘clean’ and ‘dirty’ volatilities. It can be seen that the ‘dirty’ volatility is higher than the ‘clean’ volatility.
The ‘dirty’ price is simply the sum of the ‘clean’ price and the accrued interest, so the additional volatility must arise from the accrued interest (as can be seen by the spikes at the coupon dates in the ‘dirty’ volatility in Figure 2.

Accrued interest is not a source of uncertainty – in fact it is completely certain. It is independent of the traded market yield and we know with certainty what the accrued interest will be for any settlement date. It is therefore not a source of volatility, and should not be included in this calculation. Hence, ‘dirty’ volatility should never be used.

In addition, the ‘clean’ price volatility has an almost 100% correlation with the ‘yield’ volatility, while the ‘dirty’ price has a 90% correlation.

<table>
<thead>
<tr>
<th>Bond name</th>
<th>Nominal</th>
<th>Strike yield</th>
<th>Riskfree rate</th>
<th>Expiry date</th>
<th>Expiry date for settlement</th>
</tr>
</thead>
<tbody>
<tr>
<td>R153</td>
<td>1 000 000</td>
<td>11.95%</td>
<td>11%</td>
<td>2 August 2002</td>
<td>7 August 2002</td>
</tr>
</tbody>
</table>
In the above example the potential differences in option valuation from using different volatilities are shown (using the same data as for calculating the volatilities). The option values differ by as much as 30%.

2. Bond Option Pricing

When calculating the bond’s forward price and the strike price at the option maturity, the ‘dirty’ price should be used, since the ‘clean’ price is not an actual market traded price.

When setting the strike and spot price parameters in the pricing formula, these need to be consistent. If the options have a physical settlement when exercised, then the ‘dirty’ price needs to be used for both.

3. Conclusion

It seems clear that if one is calculating volatility from bond prices, the ‘clean’ price should be used. However, for the strike price and spot price inputs, the ‘dirty’ price should to be used in the calculation, since this is the actual market price, and is the actual value that will have to be settled at exercise.