

Recovery Rates

Introduction & Overview	2
1. The Importance of Recovery Rates	3
2. Empirical Studies	4
Almost everything you wanted to know about recoveries on defaulted bonds	4
Managing loan loss on defaulted bank loans: a 24-year study	5
Bank loan Loss Given Default	6
3. Modelling of Recovery Rates	8
Estimating the Unsecured Recovery Rate (URR)	8
Estimating the Collateralised Recovery Rate (CRR)	10
Example of CRR	11
Back-testing and implementation of the CRR	12
4. Capital Relief on Collateral under the proposed Basel Capital Accord	13
Overview of the New Capital Accord	13
Collateral	14
5. The Way Forward	18
6. References/Acknowledgements	19
7. Glossary of Terms	20

Introduction & Overview

Recovery rates play a critical role in the estimation and pricing of credit risk, and yet to date this has been a neglected area – both in terms of financial modelling as well as in terms of empirical research.

This paper covers some of the major areas: it covers the major empirical results, the most common modelling approaches (as well as a suggested Emerging Markets extension), and the impact of the proposed new regulatory requirements in respect of recoveries.

It concludes with a discussion of what banks could (or should) be doing to get their recovery rate estimation to a reasonable level. We consider that loss estimations can be easily improved by a large factor with far less effort than is deployed in refining other metrics that have far less impact on the total risk banks face. We are delighted to see the advent of commercial products aimed at this topic, such as MoodysKMV's LossCalc, which we feel will become a substantial product when enhanced with the results of KMV's years of research into market-implied information.

This document, which is designed to be a kick-off point to accompany an interactive workshop, is not intended to be the even the first word in this area, but to be a start to building a cohesive body of knowledge and data on this important topic. Parts of the document may even be contradictory! We welcome any comments.

1. The Importance of Recovery Rates

Recovery rates are an often-neglected aspect of credit risk analysis. The emphasis on validating the accuracy of default probabilities and default correlations often obscures their importance, but a couple of simple examples serve to highlight the effect that even a first order improvement in estimation can make:

Example: Expected loss on a standalone loan

Assuming an exposure of R100, an EDF of 0.1, and a recovery rate of 50%. Then,

$$\begin{aligned}\text{Expected loss} &= \text{exposure} \times \text{EDF} \times \text{LGD} \\ &= \text{R}100 \times 0.1 \times 0.5 \\ &= \text{R}5\end{aligned}$$

Recovery rates aren't very accurately estimated, so to adjust the recovery rate down to 30%, resulting in a new expected loss as follows:

$$\begin{aligned}\text{Expected loss} &= \text{R}100 \times 0.1 \times 0.7 \\ &= \text{R}7\end{aligned}$$

This is an increase of approximately 40%. Achieving an identical proportional change would require adjusting the EDF upwards to 0.14, an estimate adjustment requiring basis point-level estimation precision.

Since organisations aim to price and provide for expected loss, this result illustrates the dramatic impact that a first order improvement in recovery rate estimation can have on the performance of an organisation, by the resultant improvements in risk estimation and pricing.

A magnified version of this result can be achieved by minor adjustments to LGD distribution estimation, as described and illustrated in Section 3.

2. Empirical studies

This section is a summary of some of the empirical research on recovery rates. What is immediately obvious is the lack of a standard methodology, of significant data points, and the inconsistency of results. Another obvious drawback with this research is that it is all US-based.

While aspects of the research are interesting, we feel it is at best a background of how the problem has been tackled in the past, in a developed country. While the current focus on credit risk will probably lead to a proliferation of studies over the next few years, which may yield quality empirical results, the data currently available is a very shaky basis to calibrate current pricing and risk models.

Almost everything you wanted to know about recoveries on defaulted bonds

(Altman & Vishore, Stern School NYU)

This is the base paper on defaulted corporate bonds over the period 1971-1995. It came up with a wide range of differing recovery rates, by industry, as shown below.

Industry	Average Recovery Rate %	Number of Observations
Public Utilities	70.5	56
Chemical, petroleum, rubber and plastic products	62.7	35
Machinery, instruments and related products	48.7	36
Services – business and personal	46.2	14
Food and kindred products	45.3	18
Wholesale and retail trade	44.0	12
Diversified manufacturing	42.3	20
Casino, hotel and recreation	40.2	21
Building material, metals and fabricated products	38.8	68
Transportation and transportation equipment	38.4	52
Communication, broadcasting, movie production,	37.1	65

printing and publishing		
Financial institutions	35.7	66
Construction and real estate	35.3	35
General merchandise stores	33.2	89
Mining and petroleum drilling	33.0	45
Textile and apparel products	31.7	31
Wood, paper and leather products	29.8	11
Lodging, hospitals and nursing facilities	26.5	22
Total	41.0	696

Managing loan loss on defaulted bank loans: a 24-year study

(Asarnow & Edwards, Risk Management Associates/Citibank)

This RMA-Citibank study derived a definition of economic loss in the event of default (LIED) and analysed Citibank data over a 24-year period using this definition, within 2 brand categories of loans (Commercial & Industrial and Structured)

A simplified definition for calculating the economic cost of a default (LIED), expressed as a percentage of the initial default amount (nominal):

LIED	Components
<i>W</i>	Dollar amount of each write-off taken subsequent to default
<i>ID</i>	Interest drag – the total dollar amount of forgone interest on the outstanding default balance based on a monthly present value
<i>IC</i>	Present value calculated to the date of default
<i>R</i>	Dollar amount for each unanticipated principal recovery
<i>MSC</i>	Dollar amount of other income (or expense) events
<i>IDA</i>	Initial dollar amount for the default

The components comprise the LIED (which is synonymous with Loss Given Default, LGD), as follows:

$$LIED = 100 \times \frac{W + ID - IC - R - MSC}{IDA}$$

A neglected element of this and all other definitions of LGD is the time element: traditional credit measures are usually time-insensitive, in that exposures tend to be considered in terms of their nominal values. LGD percentages typically have as numerator the preset value of all recoveries, and as denominator the value of the risky instrument, although this is often “fudged”, using the amount draw down.

Results (861 defaults)

Unlike some other studies, there was no statistically significant relationship between size of loan and LGD, but some of the individual components of LGD vary significantly by the size of the default. In particular, interest drag is a much larger part of large defaults than small ones. The most important other results are listed below:

	Commercial & Industrial	Structured
Average LIED	34.79%	12.75%
Write-offs (% of nominal)	30.88%	8.79%
Interest Drag (% of nominal)	10.73%	7.41%
Recoveries (% of gross write-offs)	13%	24%

Bank loan Loss Given Default

(Gupton, Moodys)

This paper studied 121 defaults. It compared bond values prior to default with their value the month after default. The mean bank loan value in default is 69.5% for Senior Secured and 52.1% for Senior Unsecured. The lowest 10th percentiles of recoveries were at 39.2% and 5.8% respectively. Secured claims settled more quickly than unsecured loans: 1.3 versus 1.7 years respectively. The best predictor of resolution time is the original market perception of resolution value. Recovery value that is more distant from the average (either higher or lower) strongly suggests a more rapid resolution.

Moody's found that the LGD for Senior Unsecured loans could be materially different according to the number of loans outstanding to the defaulting borrower. For single-loan defaulters, the recovery rate is 63.8% whereas for multiple-loan defaulters, the recovery rate is just 36.8%. Like the RMA-Citibank paper, Moody's found the LGD experience by broad industry groups not to differ statistically significantly from one another. They speculate that LGD correlations over time may be causing other studies to show patterns of industry-level LGD differences. A time series of the 12-month trailing default value average showed a 0.78 correlation between the loss experiences of defaulted Senior Secured bank loans versus Senior Secured public debt. This is a systematic rather than a name-specific result.

3. Modelling of Recovery Rates

For us, the implication of the previous section is that banks should develop recovery models for internal usage, to bridge the current data gap. The empirical analysis currently available should best be used as input to banks' internal modelling and data storage process rather than being directly used as inputs to pricing and risk models.

There has been very little published research on modelling recovery rates. We have taken as a base the Black-Scholes model, and a Bank of Finland paper that examined the effect of the firm's asset volatility on the recovery rate. We extend the argument to include the effects of the potential collateral value as well as the correlation of the collateral value and the asset value of the company. We show how this could be used, by dividing collateral into various classes, and for various correlations between the value of collateral assets and the value of the firm, as well as for various levels of collateral cover, to derive LGDs. The resultant impact of these collateral types on pricing is emphasised by extending the example, using the Duffie-Singleton pricing methodology.

Estimating the Unsecured Recovery Rate (URR)

By applying the Black-Scholes model to equity and debt instruments, we can write the value of a put as:

$$P_0 = -N(-d_1)V_0 + Fe^{-rT}N(-d_2) \quad (\text{A})$$

where

P_0 is the current value of the put;

V_0 is the present value of the firm's assets;

r is the risk-free interest rate;

Fe^{-rT} is the present value of the debt obligation at maturity;

$N(\cdot)$ is the cumulative standard unit normal distribution; and

$$d_1 = \frac{\ln\left(\frac{V_0}{F}\right) + (r + 0.5s^2)T}{s\sqrt{T}}$$

$$d_2 = d_1 - \mathbf{s}\sqrt{T}$$

where

\mathbf{s} is the asset sigma and

T is the interval until debt is paid back.

By purchasing the put for P_0 , the bank buys an insurance policy whose premium is the discounted expected value of the expected shortfall in the event of default. So equation (A) can be written as:

$$P_0 = \left[-\frac{N(-d_1)}{N(-d_2)}V_0 + Fe^{-rT} \right] N(-d_2) \quad (\text{B})$$

Equation (B) decomposes the premium on the put into three factors. Firstly, the absolute value of the first term inside the bracket is the expected discounted recovery value of the ban, conditional on $V_T \leq F$. It represents the risk-neutral expected payment to the bank if the firm is unable to pay the full obligation F , at time T .

The second term in the bracket is the current value of a risk-free bond promising a payment of F at time T . Hence the sum of the two terms inside the brackets yields the expected shortfall in present value terms, conditional on the firm being bankrupt at time T .

The final factor that determines P_0 is the probability of default, $N(-d_2)$. By multiplying the probability of default by the current value of the expected shortfall, we derive the premium for insurance against default.

To calculate the URR, is it necessary to calculate $\frac{N(-d_1)}{N(-d_2)}$ from the first term inside

the bracket in equation (B). Although the default probability $N(-d_2)$ and asset sigma \mathbf{s} for a company aren't readily available, the KMV technology provides both measures, and there are other approaches to obtaining these inputs. From this, d_1 and the unsecured recovery rate (URR) can be calculated.

Beware!

This is a simplistic firm model in that it assumes inter alia that the market value of assets is normally distributed and there is only one level of debt seniority.

This version of the model is therefore not robust enough for reliance in an operational context, although it can serve as a base for identifying weakness that need to be covered by further empirical research.

Estimating the Collateralised Recovery Rate (CRR)

In determining the CRR on debt, the debt is divided into two parts: the Unsecured Debt and the Collateralised Debt. The CRR is then calculated as a weighted average between the Recovery Rate on Unsecured Debt (RRUD) and the Recovery Rate on Collateralised Debt (RRCD).

Thus,

$$\begin{aligned} \text{CRR} &= \text{Weighted RRUD} + \text{Weighted RRCD} \\ &= \left(\frac{\text{Max}\{D - C_v, 0\}}{D} \cdot \text{URR} \right) + \frac{C_v}{D} \cdot (\rho(\text{URR}) + (1 - \rho) [1 - \text{VaR}(C_v | \text{Default})]) \\ &= \left(\frac{\text{Max}\{D - C_v, 0\}}{D} \cdot \text{URR} \right) + \frac{C_v}{D} \cdot (\rho(\text{URR}) + (1 - \rho) [1 - s_{C_v} \cdot \text{EDF}]) \end{aligned}$$

where:

D is the debt value;

C_v is the collateral value;

s_D is the asset volatility;

s_{C_v} is the annual collateral volatility;

EDF is the annual default probability; and

ρ is the correlation between asset value and collateral value

Example of Collateralised Recovery Rate

The correlation inputs in the examples above have not been estimated. A range of correlations is shown to show the effect on the recovery rate.

Product	Cash	Guarantee	Property	Equity	Unsecured
Collateral Value to Loan Ratio	100.00%	100.00%	100.00%	100.00%	0.00%
Asset Volatility (1)	30.00%	30.00%	30.00%	30.00%	30.00%
Annual Collateral Volatility (2)	30.00%	35.00%	40.00%	100.00%	30.00%
Correlation between (1) and (2)	0.00%	45.00%	50.00%	90.00%	100.00%

EDF	Collateralised Recovery Rate				
0.02%	99.99%	68.83%	65.37%	37.67%	30.75%
0.05%	99.99%	69.85%	66.50%	39.71%	33.01%
0.10%	99.97%	70.69%	67.44%	41.42%	34.92%
0.50%	99.85%	73.04%	70.05%	46.22%	40.30%
1%	99.70%	74.23%	71.38%	48.74%	43.15%
2%	99.40%	75.52%	72.83%	51.62%	46.46%
5%	98.50%	77.34%	74.90%	56.11%	51.79%
10%	97.00%	78.67%	76.44%	60.19%	56.88%
15%	95.50%	79.32%	77.23%	62.92%	60.47%
20%	94.00%	79.68%	77.70%	65.06%	63.40%

The Duffie-Singleton analysis shows the impact of the recovery rate on the spread. The risky rate of interest may be written as the sum of the risk-free rate and a spread term equal to the product of the default rate (I) and the proportionate loss on default ($1-f$).

$$R = r + I(1-f)$$

$$= r + s$$

The following example shows the importance of the recovery rate in the calculation of the spread. Take a company with an annual default probability of 0.50%. Extending the example used above, the recovery rate for that category of default rate is calculated over a range of 5 different collateral types. The following calculations show the impact of the recovery rates on the spread:

Product	Default Probability (1)	Recovery Rate	Loss given Default (2)	Spread (1)*(2)
Cash	0.50%	99.85%	0.15%	0.00075%
Guarantee	0.50%	73.04%	26.96%	0.13481%
Property	0.50%	70.05%	29.95%	0.14976%
Equity	0.50%	46.22%	53.78%	0.26891%
Unsecured	0.50%	40.30%	59.70%	0.29851%

Back-testing and implementation of the CRR

Back-testing the CRR is not possible at the moment due to the non-existence of recovery rate data. A possible method of back-testing is to divide the collateral assets in, say 5 categories – for instance Cash, Guarantee, Property, Equity and Unsecured. In doing so, estimates for correlations between the asset value of the company and the collateral group can be calculated. In getting some historical data the model can be calibrated to be more representative of actual recoveries.

The back-testing method can also be applied to the implementation of the CRR.

4. Capital Relief on Collateral under the proposed Basel Capital Accord

Overview of the New Capital Accord

The new accord proposes to move banks, in an evolutionary way, from a reasonably simple standardised approach to credit capital adequacy to a more sophisticated approach, where the (more granular) capital charge for each of six exposure classes will depend on a set of risk components.

The Basel proposal regime defines two broad approaches to capital adequacy:

- The standardised approach and
- The internal ratings-based (IRB) approach.

Within the IRB approach to corporate exposures are two sub-classes:

- The foundation approach, where the regulator determines many of the inputs, and
- The advanced approach, where the bank itself determines the values of the inputs by way of values and rules.

In the standardised approach, estimates of PD, LGD and in some cases maturity are used to map exposures into one of five risk weightings, whereas in the IRB approach much finer granularity is achieved by separate estimation of each of these inputs which are then used to determine a risk weighting.

To calculate risk-weighted assets, the risk weighting is multiplied with the estimated value of the exposure (hence the importance of UGD), and all such numbers are summed. The resultant figure for total risk-weighted assets may be adjusted by a standard supervisory index, effectively adjusting for the amount of concentration in the non-retail portfolio.

In contrast, the IRB approach for retail credit exposure does not have a foundation approach: opting for the IRB approach requires banks to provide estimates of either

EPD and LGD, or merely of EL, per product/client segment. Almost all of the following discussion refers to the Basel proposals for corporate credit risk.

It is envisaged that the majority of reasonably sophisticated banks will opt for the foundation approach initially, with some having the ultimate objective of migrating over time to the advanced approach. Under all of the approaches, collateral is recognised as a legitimate credit risk mitigation technique and hence certain forms of collateral may be acceptable for some capital relief. A brief description of the types of collateral and the amount of capital relief that is proposed is provided hereunder, since regardless of the risk management implications, the regulatory treatment of collateral is a significant factor.

The IRB approaches have as a key input the LGD of every exposure. Under the Foundation approach the regulator, via a set of supervisory rules, including collateral, provides this value, whereas under the advanced approach these values have to be determined internally. As with the collateral issue, the regulatory view is important to the risk management perspective, and is also provided hereunder.

Collateral

Banks may seek capital relief if their transactions are satisfactorily collateralised. For capital relief to be granted, various requirements (mainly related to legal certainty, low correlation with exposure, and a robust risk management process) must be satisfied.

There are 2 proposed treatments of collateralised transactions with the Standardized approach: the comprehensive and simple approaches. The comprehensive approach focuses on the cash value of the collateral taking into account its price volatility. The basic principle is to reduce the underlying risk exposure by a cautious measure of the value of collateral taken. Partial collateralisation will therefore be recognized. This is also the approach that will be recognized within the Foundation IRB approach. The simple approach, developed for banks that only engage to a limited extent in collateral transactions, maintains the approach of the current Accord, whereby the collateral issuer's risk weight is substituted for that of the underlying obligor. Partial

collateralisation will also be recognized, but for collateral to be recognized, it must be pledged for the life of the exposure and must be marked to market at least every six months.

Overall, the simple approach will generate higher capital requirements on collateralised transactions than the comprehensive approach.

Eligible collateral:

- Cash on deposit with the lending bank
- Government securities rated BB- (investment grade) and above
- Other securities rated BBB- and above
- Equities included in a main index (plus potentially others)
- Gold

This is a proposal aimed primarily at first world countries with high sovereign ratings. Presumably these rating criteria will be relaxed in countries with lower sovereign ratings, such as South Africa.

Mechanism

A haircut based on the estimated price volatility is calculated, and the risk weighting appropriate for the transaction is calculated, taking the haircut into account. If there is daily re-margining, the holding period will be assumed to be ten business days, whereas if the collateral is for secured lending and the collateral is marked to market daily, then the holding period will be twenty business days. Where the collateral is marked to market less frequently than daily, the haircut will be increased according to a different formula.

An example of the standardised supervisory haircuts for the 10-day holding period case is laid out below. These are the 10-business day haircuts assuming daily mark-to-market and daily re-margining to be applied in the standardised supervisory haircuts approach, expressed as percentages of the market value:

Issue Rating	Residual Maturity	Sovereigns	Banks/Corporates
AAA/AA	≤ 1 year	0.5	1
	>1year, ≤ 5 years	2	4
	> 5 years	4	8
A/BBB	≤ 1 year	1	2
	>1year, ≤ 5 years	3	6
	> 5 years	6	12
BB	≤ 1 year	20	
	>1year, ≤ 5 years	20	
	> 5 years	20	
Main Index Equities		20	
Other Listed Equities		30	
Cash		0	
Gold		15	
Surcharge for FX Risk		8	

The formula to be used in determining the collateral value is:

$$C_A = \frac{C}{1 + H_E + H_C + H_{FX}}$$

where

H_E is a haircut appropriate to the exposure (E);

C is the current value of the collateral received;

H_C is a haircut appropriate for the collateral received;

H_{FX} is a haircut for currency mismatch; and

C_A is the adjusted value of the collateral.

The risk-weighted assets for a collateralised transaction are:

$$r \cdot E = r \times \text{Max}\{E - (1 - w) \cdot C_A, w \cdot E\}$$

where

r^* is the risk weight of the position taking into account the risk reduction from the collateral; and

w is the floor capital requirement.

LGD Estimates under the proposed Basel Capital Accord

Under the foundation IRB approach, LGD is estimated through the application of standard supervisory rules. These differentiate the level of LGD based on the characteristics of the underlying transaction, including the presence and type of collateral. Basel proposes using a 50% LGD for senior un-collateralised debt, and 75% for subordinated debt. For transactions with qualifying financial collateral, the LGD is scaled using the haircut methodology similar to that used for the standardised approach, as illustrated in the table below. Non-financial collateral has a separate set of rules for determining the LGD.

For the advanced IRB approach, the bank itself determines the appropriate LGD to be applied to each exposure, on the basis of robust data and analysis. Such values would be expected to represent a conservative estimate of long-run estimates. Such motivation requires both a validation and usage component, and for the first two years cannot be less than 90% of the foundation approach. Risk weights will be scaled up or down based on these factors as well as on the maturity of the exposure.

	Condition	Effective LGD
Case 1	$C/E \leq 30\%$	50%
Case 2	$C/E > 140\%$	40%
Case 3	$30\% < C/E \leq 140\%$	$\left[1 - \left(0.2 \cdot \frac{C/E}{140\%} \right) \right] \times 50\%$

5. The way forward

While the data issue will remain problematic for the foreseeable future, the operational benefits of even a basic approach to modelling recovery rates makes it worthwhile to devote some time to the problem. We suggest that South African banks adopt a multi-pronged yet pragmatic approach. The following are the list of steps we suggest are adopted:

- Establish an internal definition of loss that is as “economic” as possible, and yet reasonably easy to estimate (ex ante) and obtain (ex ante). This could take the form of something like the RMA-Citibank definition discussed earlier. You may attempt to backfill with previous defaults, but this is highly unlikely to be successful. This definition should then serve as the basis for estimation and database design.
- Build a robust process that ensures that future defaults successfully populate a recoveries database. Use this information judiciously, together with other industry data (for example if the Central Bank develops a central source of recovery data, or any other industry information is distributed).
- Ensure that current and potential future exposure, EDF, and collateral information is captured and valued as accurately as possible.
- Build a recovery rate model that is theoretically plausible, but uses inputs you can obtain or reasonably estimate. This may be based on something like our model presented earlier, or on something more basic. It could even be based initially on the estimates of experienced credit personnel. Estimate recovery rates for every exposure, and parameterise your loss distribution as accurately as possible.
- Use both your internal estimates of recovery, as well as the proposed regulatory version, in estimating your losses as well as your projected risk and regulatory requirements. Understand the differences between your internal estimates and those of the regulator, particularly with respect to collateral.
- Once you feel your estimates are reasonable, implement them in your pricing and risk capital allocation policies.

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7. Glossary of Terms

- EL: Expected Loss
- EDF: Expected Default Frequency (KMV's PD)
- EDP: Expected Default Probability (Moody's PD)
- IED: Exposure in the Event of Default (UGD)
- IRB: Internal Ratings-based Approach
- LGD: Loss given Default. Sometimes referred to as Loss in the Event of Default (LIED). Typically expressed as a proportion of nominal. Equivalent to $(1 - \text{Recovery Rate})$.
- PD: Probability of Default (EDF). Is independent of time, and can have several variants (i.e. actual, forward, cumulative), but usually refers to the 1 year rate.
- UGD: Usage Given Default (IED). Typically expressed as a percentage of committed lines.
- UL: Unexpected Loss. This can be either a measure of volatility of expected loss, or a capital-oriented measure of some of the loss distribution tail.