Introduction to Yield Curves

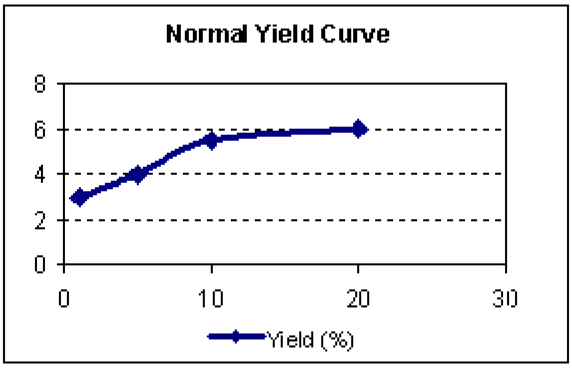
1. Introduction

A yield curve is a graphical representation of where interest rates are today. There is not one single interest rate; there is an interest rate for those who want to invest now for the next three months, there is another interest rate for those who want to invest now for the next six months, and so on out to thirty years and longer. The yield curve shows the rate of return that can be locked in now for various terms into the future. The left, vertical axis of the graph shows the annual percent yield that is currently available in the marketplace. The bottom, horizontal axis shows the investment holding period. The yield curve is a line connecting the rates of return that can be earned to the time periods that can be committed to.

Suppose an investor could choose from the following returns for their respective investment holding periods:

Yield (%)	Holding Period (years)
3.0	1
4.0	5
5.5	10
6.0	20

At this point in time, an investor could lock in a one-year return of 3%. By locking in right now for the next five years, he could lock in an annual return of 4%. Further, an investment in a 10-year bond offers a 5.5% return per year right now, and 20-year bonds are currently returning 6% per annum. The current yield curve is found by simply graphing these data points.



The investment instruments used to construct the yield curve should all have the same risk, the same features and the same coupon rate. As a result, the only difference in the returns that can be earned should be attributable to the holding period. Government bonds are often used to construct yield curves, because:

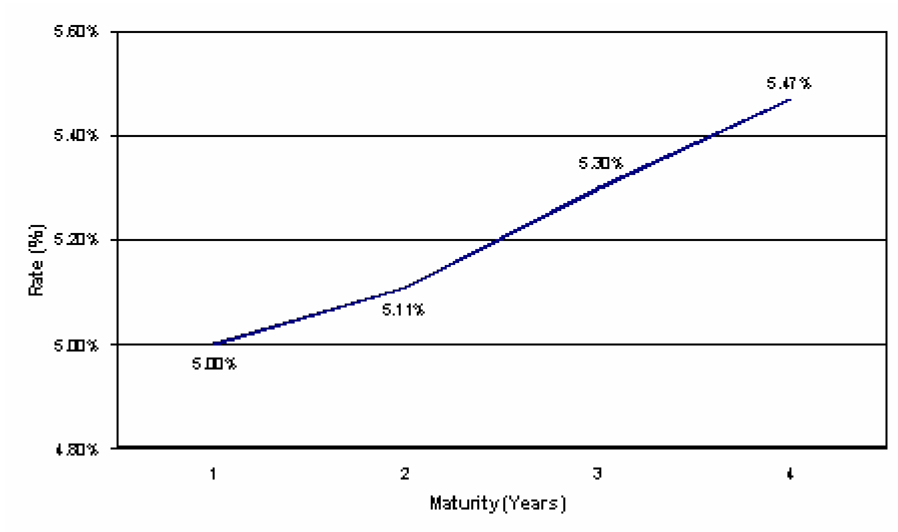
1. They have a broad range of maturities available;
2. All maturities have the same credit risk as each other; and
3. Differences in coupon rates are nullified by using stripped (no coupon) versions of the various bonds.

In theory, for each future payment of a coupon security, there exists a zero coupon rate that discounts the payment to its present value. These rates constitute the zero coupon yield curve, points along which represent the yield to maturity of a zero coupon bond for the appropriate maturity rate. It is impossible to estimate the zero coupon curve from the existing par bond yield curve.

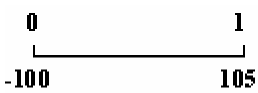
The zero coupon rates are calculated using an interactive methodology whereby the zero coupon rate is determined from a known yield curve for the successive points in time (often referred to as "bootstrapping").

2. Bootstrapping the Zero Curve

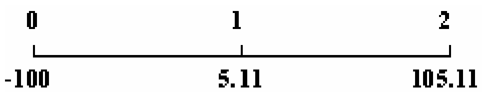
Consider a 4-year security with annual payments where the curve is as follows:



The rates above give us a par bond yield curve:

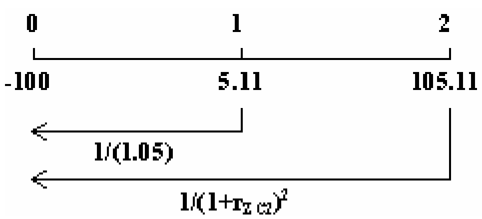


r_{Y1} = 0.05



r_{Y2} = 0.0511

where r_{Yi} is the yield on a par bond that matures at time i. From this, we can compute zero-coupon rates. Cash flows that occur at time 1 should be discounted at 5%. At what rate should the cash flows at time 2 be discounted so that r_{Y2} = 0.0511?



r_{ZC2} is the zero coupon rate for the two year maturity. This implies:

$$100 = \frac{5.11}{1.05} + \frac{105.11}{(1 + r_{ZC2})^2}$$

so r_{ZC2} = 5.113%.

Repeating for the third zero coupon yield:

$$100 = \frac{5.30}{1.05} + \frac{5.30}{(1.05113)^2} + \frac{105.30}{(1 + r_{ZC3})^3}$$

so r_{ZC3} = 5.312%.

These rates can be used as discount rates for cash flows occurring at the corresponding times.

3. Forward Rates

We can also imply forward rates from these zero-coupon rates. Consider investing your money for 2 years. You can either lock in the 2-year rate (5.113%) or invest for one year (5%) and lock in the forward rate ($r_{1,2}$) for the second year. Arbitrage should make both returns identical:

$$(1 + r_{ZC2})^2 = (1 + r_{ZC1}) \cdot (1 + r_{1,2})$$

where $r_{1,2}$ is the forward rate for an investment beginning at time 1 and ending at time 2. In the case above:

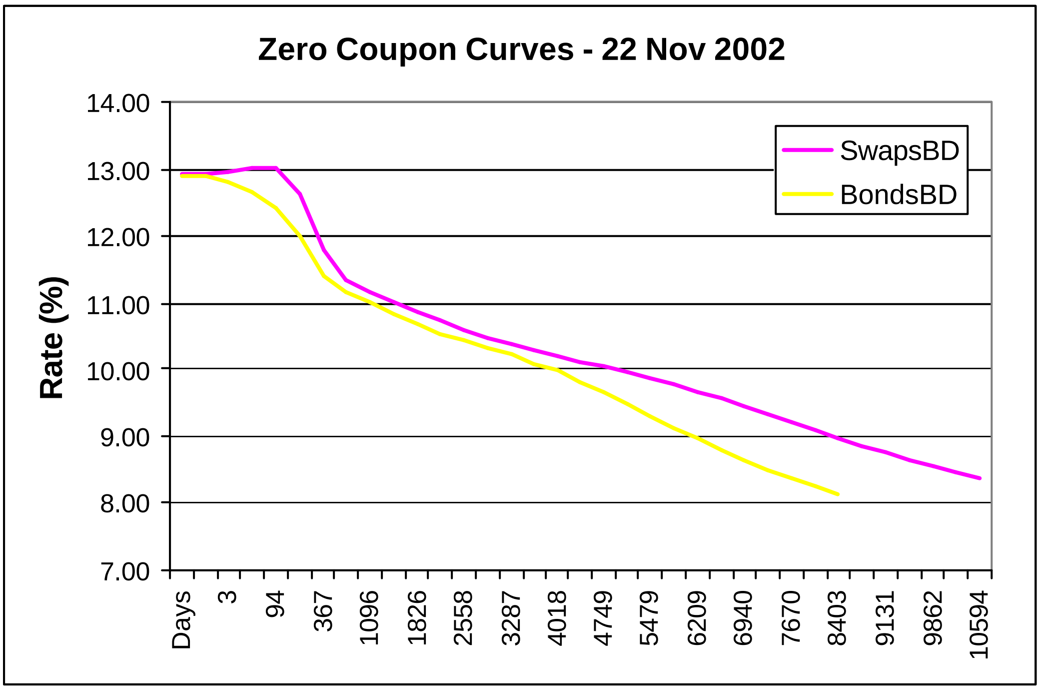
$$(1.05113)^2 = (1.05) \cdot (1 + r_{1,2})$$

meaning that $r_{1,2}$ = 5.226%. This can easily be repeated for the forward rates $r_{2,3}$ and $r_{3,4}$.

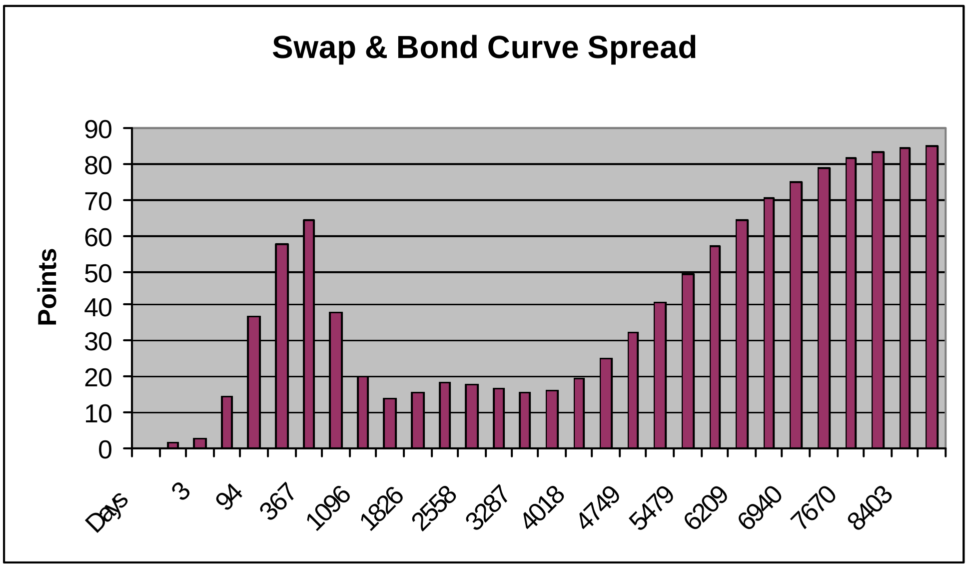
Maturity	Curve	Year 1	Year 2	Year 3	Year 4
1	5.00%	100.0000	4.8667	5.0476	5.2095
2	5.11%		95.1333	4.7969	4.9508
3	5.30%			90.1554	4.6833
4	5.47%				85.1564
Par		100	100	100	100
ZC Rates		5.0000%	5.1128%	5.3122%	5.4940%
Forwards		5.0000%	5.2257%	5.7122%	6.0414%

Because the cash flow dates of the instruments to be valued rarely exactly match the dates for which zero curve points have been developed, interpolation between data points is needed to solve the problem. Various methods are used, including linear interpolation, exponential interpolation and cubic splining.

4. BEASSA Zero Coupon Curves



The chart above represents the BEASSA Zero Coupon curves for 22 November 2002. The Swap curve is consistently above the Bond curve. This indicates the credit spread inherent in the Swap curve and the fact that the likelihood of default is lower with government bonds.



The above chart is a representation of the points spread between the Swap and Bond Zero Curves for 22 November 2002.